Heuristic Based Optimal Path Planning for Neurosurgical Tumor Ablation

Ajeet Wankhede¹, Likhita Madiraju¹, Dipam Patel¹, Kevin Cleary², Chima Oluigbo³, Reza Monfaredi²

¹Robotics Masters Program, A. James Clark School of Engineering, University of Maryland, College Park, Maryland
²Sheikh Zayed Institute for Pediatric Surgical Innovation, Children's National Medical Center, Washington, DC
³Neurosurgery Department, Children's National Medical Center, Washington, DC
ajeet@umd.edu¹, likhita.madiraju@gmail.com¹

ABSTRACT

In brain tumor ablation procedures, imaging for path planning and tumor ablation are performed in two different sessions. Using pre-operative MR images, the neurosurgeon determines an optimal ablation path to maximize tumor ablation in a single path ablation while avoiding critical structures in the brain. After pre-operative path planning the patient undergoes brain surgery. Manual planning for brain tumor ablation is time-intensive. In addition, the pre-operative images may not precisely match the intra-operative images due to brain shift after opening the skull. Surgeons sometimes therefore adjust the path planned during the surgery, which leads to increased anaesthesia and operation time. In this paper, a new heuristic-based search algorithm is introduced to find an optimal ablation path for brain tumors, that can be used both pre- and intra-operatively. The algorithm is intended to maximize the safe ablation region with a single path ablation. Given the tumor location, healthy tissue locations, and a random start point on the skull from medical images, our proposed algorithm computes all plausible entry points on the skull and then searches for different ablation paths that intersect with the tumor, avoids the critical structures, and finds the optimal path. We implemented Breadth First Search (BFS), Dijkstra, and our proposed heuristic based algorithms. In this paper we report the results of a comparative study for these methods in terms of the search space explored and required computation time to find an optimal ablation path.

Keywords—neurosurgery, path planning, tumor ablation, breadth first search, Dijkstra, heuristic cost, heuristic based search algorithm

1. INTRODUCTION

An estimated 80 thousand new cases of brain and other central nervous system (CNS) tumors are expected to be diagnosed in the United States in 2018 ¹. There are several medical procedures available for the treatment of tumors including medication, open brain surgery, and thermal ablation (laser and RF). In thermal ablation, general anesthesia is first administered. A frame is then placed over the head to help the surgeon insert a probe in the best place to reach the tumorous lesion. The surgeon requires time to perform path planning and find entrance angles of the ablation electrode to reach the tumor using MRI scans in two views (sagittal and coronal) ². Surgical virtualization or neuronavigational technology can be used in brain surgery to visualize and plan for the surgery scenarios in a 3D virtual model. With this, the surgeon can simulate the surgical procedure for a number of permutations of probe orientation and incision points while avoiding critical structures. After successful pre-operative path planning, the patient is prepped for the surgery, and the electrode is placed through a small incision in the scalp in the operating room with a sub-millimeter accuracy. After placing the electrode, the patient is moved into an MRI scanner, and the surgeon verifies the placement of the tip of the electrode and performs ablation with real-time visualization of the ablation region under MRI guidance.

Manual path planning for tumor ablation is time consuming, subjective, and depends heavily on the experience of the surgeon. Brain shift could result in a targeting error, necessitating path planning adjustments by the surgeon during the surgical procedure. Moreover, MRI is a relatively expensive imaging modality. After the electrode insertion, most thermal lesion ablations in the brain can be made in a duration of 30 to 90 seconds ² while the total time under anesthesia
is about 4 hours. This means the careful setup and electrode placement before ablation treatment takes the longest time. Increased anesthesia time can lead to serious consequences especially for pediatric and geriatric populations.

In this paper we propose a novel technique that eventually could be integrated with an MRI-compatible robotic system for robotically-assisted brain interventions. There are various image segmentation techniques like k-means, fuzzy c-means, region growing algorithms, and curvelet transform used for detection of tumor. The robotic system, Pathfinder (Prosurgics, formerly Armstrong Healthcare Ltd.), has been cleared by the FDA for neurosurgery (2004). Using the system, the surgeon specifies a target and trajectory on a pre-operative medical image, and the robot guides the biopsy electrode during the procedure. A research team conducted a feasibility study on an algorithm to provide a surgeon with a list of safe entry points from MRI scans using a cost function defined by the Euclidian distance of a straight probe from critical structures. However, this technique does not take into account electrode and tumor orientations. Thus, this algorithm is unfavorable for elongated tumors which would need multiple incisions. Further, the surgeon is still required to analyze the output data to select an optimal path. Therefore, there exists a need for an effective path planning algorithm to minimize the search space and search time to determine an optimal ablation path.

Here we propose a heuristic based method that searches for an optimal probe orientation and entry point on the skull which produces the maximum ablation with a single path ablation. A smaller search space is explored, hence reducing the computational time. Figure 1 shows a hypothetical optimal path for ablation which is along the longitudinal axis of an elongated cancerous lesion.

![Figure 1 A schematic of brain’s structure depicting the possible biopsy path and optimal ablation path](image)

**2. METHOD**

Based on the MR Images, the surgeon identifies the position of tumor and critical structures in the brain. The surgeon selects points approximately along the principle axis of the tumor, and points along the boundary of the obstacles. Also, the surgeon selects a starting point as an entry point through which the electrode would be penetrated inside the skull. The selected points are used to model the tumor and obstacles inside the brain. Based on our proposed method the surface of the skull will be swept starting from a random entry point and determine the optimal electrode orientation which is defined by maximum penetration through the elongated tumor, while avoiding all obstacles. Here, the obstacles represent nerves and critical structures.

In this paper simplified models are considered for skull, elongated tumor, and critical structures and for ease of calculation and path planning method evaluation. We assume the human skull to be a spheroid with principal axes a, b, and c which are defined as follows:

\[
\frac{x^2}{a^2} + \frac{y^2}{b^2} + \frac{z^2}{c^2} = 1, \quad a \neq b, \text{ and } a = c \quad (1)
\]

The state space is defined as \((\alpha, \beta, \theta, \varphi)\), where \(\alpha\) and \(\beta\) are the rotations of the electrode about the normal located at the entry point, while \(\theta\) and \(\varphi\) are defined to address the point on the spheroid as follows:
\[
x = a \cos(\theta) \cos(\phi) \\
y = b \cos(\theta) \sin(\phi) \\
z = c \sin(\theta)
\] 

The surgeon selects the start point \( P_0 = (\theta_0, \phi_0) \) on the skull surface, \( N \) points \( \{T_1, \ldots, T_N\} \) along the tumor axis and \( M \) points \( \{O_1, \ldots, O_M\} \) on the obstacle boundary. The search space is defined as:

\[
\theta \in \left[0 + \frac{\pi}{2}, \frac{\pi}{2}\right], \phi \in \left[0 + \frac{\pi}{2}, \frac{\pi}{2}\right], \alpha \in \left[0 + \frac{\pi}{2}, \frac{\pi}{2}\right], \beta \in \left[0 + \frac{\pi}{2}, \frac{\pi}{2}\right]
\]

Figure 2 represents the planning concept for this problem. The normal vector \( N_{i,j} \) to the surface at entry point \( i,j \) is shown. The planning algorithm calculates the orientation and entry point of the biopsy electrode suitable to target the cancerous lesion along its longitudinal axis. We proposed a heuristic based search algorithm in this paper which is compared with Modified Breadth First Search (BFS) and Dijkstra algorithm to evaluate its performance. The suggested heuristic function minimizes the state space domain and decreases the computational cost substantially while maximizing the ablation zone in a single path ablation. In the Modified BFS, the optimal path is determined by the maximum penetration of the electrode in the tumor. For Dijkstra’s algorithm the suggested cost function \( F \) is defined as

\[
F(\alpha, \beta, \theta, \phi) = \min_{\{A_1, A_2, \ldots, A_N\}} \frac{A_1 + \cdots + A_N}{N}
\]

where \( A_1, A_2, \ldots, A_N \) are the angles between \( L_1, L_2, \ldots, L_N \) and the tumor axial vector \( L_T \) such that \( L_i \) is the line joining the \( i \)th tumor points to the entry point. The average of these angles is considered as the cost for each entry point. The heuristic function is defined as the average normal distance between the electrode and the points on the obstacle i.e. \( \{O_1, O_2, \ldots, O_M\} \). The goal is to maximize the heuristic function so the distance between the electrode and the obstacle is maximized. By the proposed strategy, a smaller search space is explored, hence reducing the computational time.

Figure 2 Diagram of proposed search algorithm concept which searches along the skull surface with \( \Delta \theta \) and \( \Delta \phi \) steps while minimizing the angle between \( L_T \) and \( \{L_1, \ldots, L_N\} \)

### 3. MODELING IN MATLAB

Figure 3 depicts the MATLAB model for the problem presented in this paper. Red points are the tumor points along its axis and blue points are the obstacle points along the boundary given by the surgeon. For simulation purposes, five tumor points and eight obstacle points have been considered. The shapes of tumor and obstacles are approximated as cylinder and sphere. The center of the sphere is calculated as the mean of the obstacle points and the radius of the sphere is considered as the maximum Euclidean distance between the obstacle points and their mean. Any number of obstacles may be added in a similar fashion. From the given five tumor points, a tumor axis is approximated. The following steps are undertaken to fit a tumor line for the given 3D points provided by the surgeon:

a. Calculate the mean of given points
b. Subtract mean from all points
c. Form an averaged co-variance matrix
d. Compute SVD of the co-variance matrix
e. Find the eigen vector corresponding to largest eigen value. This vector will give the direction vector of line
f. Use the parametric equation of line to find the points lying on the line

From the tumor axis unit vector, a cylindrical tumor is modelled along its axis having length as the maximum Euclidean distance between the tumor points and their mean. The equation of the cylinder about an arbitrary axis with radius $r$ and length $L$:

$$\|\hat{a} \times (\hat{p} - \hat{b})\|^2 = r^2$$  \hspace{1cm} (6)

where, $a$ is the direction vector of tumor axis, $b$ is the mean of tumor points (center of cylinder) and $p$ is any point on the cylinder surface. Point $p$ is at signed distance of

$$0 \leq \hat{a} \cdot (\hat{p} - \hat{b}) \leq L$$  \hspace{1cm} (7)

from the point $b$ along the axis. As the tumor axis is not aligned with the z-axis, the x, y and z coordinates are multiplied with a rotation matrix to get the coordinates with respect to world frame. The rotation matrix is calculated using the Rodrigues formula given by:

$$R = I + v_{[x]} * \sin\theta + v_{[x]}^2 * (1 - \cos\theta)$$  \hspace{1cm} (8)

where $I$ is the identity matrix, $v_{[x]}$ is the skew symmetric matrix of vector $v$ perpendicular to the cylinder axis and the z-axis of world coordinate frame. The entry point for the search is selected as the point of intersection of tumor axis and the spheroid surface which is shown in green. This point is calculated by solving equation of line and equation of spheroid. From the two solutions, we select one with positive Z-axis coordinate. A normal to the surface at entry point is calculated by finding the gradient using equation (9). The normal line is shown in green in Figure 3.

$$\nabla f \big|_{\text{entry point}} = \left(\frac{2x}{a^2}, \frac{2y}{b^2}, \frac{2z}{c^2}\right)$$  \hspace{1cm} (9)

Figure 3 MATLAB model depicting skull, tumor, obstacle, entry point

### 4. MODIFIED BFS

The starting point is the point of intersection of tumor axis with the skull surface. From the given starting point, all the combinations of orientation ($\alpha$ and $\beta$) within the limits are computed for the specified resolution (5 degree). Then the next entry point is selected by varying $\theta$ and $\phi$ with the specified resolution (1 degree). The normal vector to the skull surface at the entry point is determined using equation (9). This procedure is repeated while $\theta$ and $\phi$ are within the limits. The action set consists of the following four actions which dictate the direction of search:
\[
\left\{ \left( \theta + \frac{\pi}{180}, \phi + \frac{\pi}{180} \right), \left( \theta + \frac{\pi}{180}, \phi - \frac{\pi}{180} \right), \left( \theta - \frac{\pi}{180}, \phi + \frac{\pi}{180} \right), \left( \theta - \frac{\pi}{180}, \phi - \frac{\pi}{180} \right) \right\} \quad (10)
\]

For each of the above actions, there exists four actions dictating the direction of orientation, namely:

\[
\left\{ \left( \alpha + \frac{5\pi}{180}, \beta + \frac{5\pi}{180} \right), \left( \alpha + \frac{5\pi}{180}, \beta - \frac{5\pi}{180} \right), \left( \alpha - \frac{5\pi}{180}, \beta + \frac{5\pi}{180} \right), \left( \alpha - \frac{5\pi}{180}, \beta - \frac{5\pi}{180} \right) \right\} \quad (11)
\]

A straight line of length 10 cm starting from the entry point, depicting the ablation electrode, is generated for each state. The Euler angles from the direction cosines of the normal are computed. The state of the electrode is defined by three components \([\cos(\omega_1), \cos(\omega_2), \cos(\omega_3)]\) where \(\omega_1\) and \(\omega_3\) are given by: \(\alpha \pm \frac{5\pi}{180}\) \(\beta \pm \frac{5\pi}{180}\) and \(\omega_2\) is the orientation of normal in the XY plane.

Those states for which the electrode is not intersecting with the tumor are eliminated. Let the equation of electrode for any point on the electrode \(p\) be

\[
\vec{p} = \text{entry point} + \vec{n}t \quad (12)
\]

where, \(\vec{n}\) is the direction vector of the electrode and \(t\) is the distance of the point \(p\) from the entry point. To find the intersection of electrode with tumor, equations (6) and (7) are solved with equation (12). Upon rearranging the terms, a quadratic equation having two real solutions \(p_1\) and \(p_2\) is obtained. Only those solutions which lie between 0 and \(L\) are the plausible electrode orientations for intersection with tumor.

These are possible solutions for the electrode intersecting the tumor whilst neglecting the obstacles. The states for which any part of the electrode is inside, or on boundary of obstacle space, are discarded by solving the equation of sphere and electrode line equation (12). For the remaining states, the penetration of the electrode inside the tumor is computed. For calculating the penetration, the Euclidean distance between the two solutions \(p_1\) and \(p_2\) is calculated. The state with maximum penetration is chosen as the solution for the optimal position and orientation of the electrode. The search is called Modified BFS as the reward function for each state is maximized. The implementation of Modified BFS is shown in Figure 4 which depicts the top view of electrode orientation passing through tumor while avoiding obstacle and thus selecting the optimal path.

The BFS algorithm requires \(\left( \pi \times \frac{180}{\pi} \right)^4 = 10^9\) iterations to find the optimal path. Therefore, for the BFS algorithm, the computational time is very high and is impractical for intraoperative path planning. The trouble is that BFS is greedy and tries to move towards the goal even if it is not the right path. In order to reduce computational time, the exploration of search space must be reduced by incorporating a cost function into the Modified BFS. This gives rise to the Dijkstra algorithm which will be discussed in the next section.
5. DIJKSTRA ALGORITHM

The search space is defined with a resolution of 1 degree for θ and φ, and 5 degree for α and β for Dijkstra\textsuperscript{11}. A cost function is introduced as specified in equation (5) which tries to orient the electrode along the tumor axis for maximum ablation zone. The angles $A_1, A_2, \ldots A_N$ are calculated as shown below:

$$A_i = \cos^{-1} \left( \frac{\mathbf{L}_i \cdot \mathbf{a}}{||\mathbf{L}_i|| \cdot ||\mathbf{a}||} \right)$$

where $\mathbf{L}_i$ is the vector joining $i^{th}$ tumor point and the entry point and $\mathbf{a}$ is the vector in the direction of the tumor axis. The average of the angles is the cost for each entry point. The element having minimum cost from the queue is considered as the current node and expanded further.

Figure 5 MATLAB model representing Dijkstra algorithm for start point $\theta_0 - \frac{\pi}{2}, \phi_0 - \frac{\pi}{2}$.

The action resulting in lowest cost is given preference, thus reducing the search space. The search is started from the boundary of search space. As explained in Modified BFS, penetration for the plausible solutions is calculated. The state with maximum penetration is then chosen as the optimal path for insertion of electrode.

Figure 5 shows the implementation of Dijkstra algorithm. The green lines represent the normal at the entry points in the explored search space starting from $\theta_0 - \frac{\pi}{2}, \phi_0 - \frac{\pi}{2}$. The black lines depict the acceptable states (electrode orientations at different entry points). The cyan colored line is the optimal path. The search continues until a cost is found which is greater than its parent node. This can be thought of as a potential field lines\textsuperscript{12} where the direction of the lines points the direction of decreasing cost. The cost at which search is terminated is regarded as a minima. This may not be an optimal solution as seen in Figure 6 where the starting point has been taken as $\theta_0 - \frac{\pi}{3}, \phi_0 - \frac{\pi}{3}$. The starting point is changed multiple times for the search and the algorithm is run until it converges to a lower cost. This is regarded as a global minima.

Figure 6 MATLAB model representing Dijkstra algorithm converging at a Local Minima.
Although the Dijkstra algorithm reduces the number of entry points (θ, φ) explored, the orientations of electrode (α, β) computed at each entry point remains the same as those explored in Modified BFS as the cost only depends upon the entry point. The suggested heuristic algorithm minimizes the state space domain further and decreases the computational cost. The heuristic based algorithm will be explained in the next section.

6. OUR PROPOSED HEURISTIC BASED ALGORITHM

The search space is defined with a resolution of 1 degree for θ and φ and 1 degree for α and β. The heuristic function is defined as the average distance of points on the obstacle from the electrode. The shortest distance between the obstacle point and the electrode line is its perpendicular distance which is computed as

\[ d = \frac{\|[(x_0-x_1) \times (x_0-x_2)]\|}{\|x_2-x_1\|} \]  

where, \( x_1 \) and \( x_2 \) are the points on the tumor line and \( x_0 \) is the point on the obstacle as shown in Figure 7.

The total heuristic cost for each state will be the sum of the individual heuristic costs defined by its average distance from each obstacle. The element having minimum cost from the queue is considered as current node and expanded further. The action resulting in lowest cost is given preference, thus reducing the search space for orientation. The search begins with the initial condition \( \alpha = 0 \) and \( \beta = 0 \) with respect to the normal. As explained in Modified Breadth First Search, penetration for the plausible solutions is calculated. The state with maximum penetration is then chosen as the optimal path for insertion of electrode.

Figure 8 shows the implementation of our proposed algorithm. The green lines represent the normal at the entry points in the explored search space starting from \( \theta_0 - \frac{\pi}{2}, \phi_0 - \frac{\pi}{2} \) similar to Figure 5 for Dijkstra algorithm. As seen from the figures, the electrode orientations for our proposed method at different entry points are less explored as compared to Dijkstra depicted by the black lines. However, both algorithms converge to the same optimal path shown by the cyan colored line. But in this method, the search space and hence computation time has reduced significantly.
As seen in Dijkstra algorithm, some entry points converge at local minima resulting in a non-optimal solution. This issue is solved to some extent with the help of our algorithm as it is feasible to have a higher resolution (1 degree) of $\alpha$ and $\beta$ compared to Dijkstra (5 degree), thus enabling an accurate action set. Comparing Figure 9 and Figure 6, it can be seen that our proposed algorithm converges at the global minima for the same start point, at which the Dijkstra algorithm converged at local minima.

7. RESULT AND CONCLUSION

The Modified BFS, Dijkstra, and our proposed algorithm are compared in terms of computational time, state space explored, and number of nodes created. For Dijkstra algorithm, the explored state space is reduced significantly (by 80%) and hence the computational time is one fifth of Modified BFS. The results of modified BFS and Dijkstra are extrapolated for a resolution of 1 degree for $\alpha$, $\beta$, $\theta$ and $\varphi$ to compare with our algorithm, as shown in Table 1. By avoiding the search of unnecessary states, our algorithm is $3 \times 10^5$ times faster than Modified BFS.

Table 1 Comparison of Modified BFS, Dijkstra and our proposed method having resolution of 1 degree for $\alpha$, $\beta$, $\theta$, $\varphi$

<table>
<thead>
<tr>
<th>Parameters</th>
<th>Modified BFS</th>
<th>Dijkstra</th>
<th>Our proposed method</th>
</tr>
</thead>
<tbody>
<tr>
<td>Number of Nodes</td>
<td>$10^9$</td>
<td>$10^8$</td>
<td>54,947</td>
</tr>
<tr>
<td>Computation Time</td>
<td>3100 hours</td>
<td>620 hours</td>
<td>40 seconds</td>
</tr>
<tr>
<td>State Space Explored (%)</td>
<td>100%</td>
<td>20%</td>
<td>0.005%</td>
</tr>
</tbody>
</table>

Our simulation results show that our method requires only 40 seconds to calculate the optimal electrode orientation and entry point in the simplified model of the brain with obstacles and a single elongated tumor. The algorithm was tested on over 50 different locations of tumor and obstacles which produced consistent results.

8. FUTURE WORK

To model the brain accurately, we plan to do quantitative analysis of MRI images to detect the tumor from their boundaries or contours, segment the tumor, area of tumor and the location of the tumor and obstacles\textsuperscript{14, 15}. The method suggested in this reference\textsuperscript{16} incorporates noise removal functions, segmentation and morphological operations using MATLAB software. This could further reduce the time as the surgeon would no longer need to select the tumor and obstacle points for the algorithm as proposed in this paper. We also plan to implement various novel heuristic functions to improve the computation time and accuracy of path planning electrode for tumor ablation.
REFERENCES


